Locality in the Everett Interpretation of Quantum Field Theory*

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Abstract

Recently it has been shown that transformations of Heisenberg-picture operators are the causal mechanism which allows Bell-theorem-violating correlations at a distance to coexist with locality in the Everett interpretation of quantum mechanics. A calculation to first order in perturbation theory of the generation of EPRB entanglement in nonrelativistic fermionic field theory in the Heisenberg picture illustrates that the same mechanism leads to correlations without nonlocality in quantum field theory as well. An explicit transformation is given to a representation in which initial-condition information is transferred from the state vector to the field operators, making the locality of the theory manifest.

Key words: quantum field theory, locality, Everett interpretation, Heisenberg picture.

1 Introduction

Bell's theorem [1] does not apply to quantum mechanics in the Everett interpretation [2]. The premises of the theorem include the implicit assumption that, each time an experiment is performed, there is a single outcome to the experiment. In the Everett interpretation, all possible outcomes occur. Thus, Everett-interpretation quantum mechanics is not demonstrated by Bell's theorem to be nonlocal [3, 4].[1]

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[1] Locality in Everett-type interpretations of quantum mechanics is also discussed in [2], [5]-[17].

It is another question, however, whether Everett-interpretation quantum mechanics *is* in fact local—that is, whether or not it provides a causal explanation for correlations observed between distant systems which have interacted in the past, such as those observed in Einstein-Podolsky-Rosen-Bohm (EPRB) experiments [18, 19]. In other interpretations of quantum mechanics the view is taken that each repetition of an experiment results in a single outcome. In the context of these other interpretations we attempt to understand how it comes about that when the experimenters, Alice and Bob, set their respective analyzer magnets to be parallel, the results which they obtain for the respective spins of a pair of singlet-state electrons are perfectly anticorrelated. Bell's theorem states that neither quantum mechanics nor any other conceivable physical theory can provide a causal explanation for them and, at the same time, account for the correlations observed when the magnets are not parallel.

In the Everett interpretation, correlations between the two experimenters' results are not at issue; rather, a different question of causation arises. According to Everett, both possible outcomes, spin-up and spin-down, occur at each analyzer magnet and, at the conclusion of the experiment, there are two copies of each experimenter.[2] When they compare their respective results using some causal means of communication, Alice-who-saw-spin-up only talks to Bob-who-saw-spin-down, and Alice-who-saw-spin-down always converses with Bob-who-saw-spin-up. What is the mechanism which brings about this perfect anticorrelation in the possibilities for exchange of information between the Alices and the Bobs?

Deutsch and Hayden [21] have identified this mechanism. In the Heisenberg picture of quantum mechanics, the properties of physical systems are represented by time-dependent operators. When two systems interact, the operators corresponding to the properties of each of the systems may acquire nontrivial tensor-product factors acting in the state space of the other system. These factors are in effect labels, appending to each system a record of the fact that it has interacted with a certain other system in a certain way [22]. So, for example, when the two particles in the EPRB experiment are initially prepared in the singlet state, the interaction involved in the preparation process causes the spin operators of each particle to contain nontrivial factors acting in the space in which the spin operators of the other particle act. When Alice measures the spin of one of the particles, the operator representing her state of awareness ends up with factors which act in the state space of the particle which she has measured, as well as in the state space of the other particle. The operator corresponding to Bob's state of awareness is similarly modified. When the Alices and Bobs meet to compare notes, it is these factors which lead to the correct pairing-up of the four of them.

The amount of information which even a simple electron carries with it regarding the other particles with which it has interacted is thus enormous. In [22] I termed this the problem of "label proliferation," and suggested that the physical question of how all this information is stored[3] might receive an answer in the framework of quantum field theory.

[2] Or, in another variant of the Everett interpretation, two continuously-infinite sets of copies of each experimenter [20]

[3] There is no *formal* problem in this regard: The mathematical elements which encode the label information, the tensor-product factors, are a straightforward consequence of the basic rules of quantum mechanics in the Heisenberg picture.

More generally, quantum field theory is a description of nature encompassing a wider range of physical phenomena than the quantum mechanics of particles; it is therefore of interest to investigate the degree to which the conceptual picture of the labeling mechanism for bringing about correlations at a distance in a causal manner accords with the field-theoretic formalism.

Indeed, there is a line of argument which leads to the conclusion that Everett-interpretation Heisenberg-picture quantum field theory *must* be local. The dynamical variables of the theory are field operators defined at each point in space, whose dynamical evolution is described by local (Lorentz-invariant, in the relativistic case) differential equations. And the Everett interpretation removes nonlocal reduction of the wavefunction from the formalism. So how can nonlocality enter the scene? As we will see, this argument as it stands is incorrect, but it can be modified so that its conclusion, the locality of Everett-interpretation Heisenberg-picture quantum field theory, holds.

For simplicity, we will consider below only nonrelativistic quantum fields of spin 1/2 fermions, and we will not explicitly include in the formalism degrees of freedom corresponding to the states of awareness of observers. In Sec. 2 below I recast the first-quantized analysis of the EPRB experiment presented in [22] into a Fock-space formalism of spinning particles with no spatial degrees of freedom. This analysis is generalized in Sec. 3 to nonrelativistic quantum field theory in three space dimensions. In Sec. 4 a unitary transformation is presented which transfers initial-condition information from the Heisenberg-picture state vector to the operators. In Sec. 5 I review the conclusions that follow from this analysis regarding the questions of label proliferation and locality.

2 EPRB Entanglement in Fock Space

To construct a Fock space representation of the particles involved in the EPRB experiment, we will first set up a suitable first-quantized Schrödinger-picture formalism, then proceed to the corresponding Fock space representation by standard methods (see, e.g., [23]-[25]).

2.1 First-Quantized Formalism

2.1.1 Single-Particle Hilbert Space

We first consider a Hilbert space for a single spin 1/2 particle which has no spatial degrees of freedom but which does posses one additional internal two-valued degree of freedom, which we will refer to as its "species." If the operator $\widehat{\rho}$ corresponds to a determination of the species of the particle, and $\widehat{\sigma}_z$ measures the z-component of spin in units of $\hbar/2$, a complete set of eigenstates for this system, $\mathcal{S}$, is

$$|\mathcal{S};\rho_{[r]},\alpha_i\rangle, \qquad r,i=1,2, \tag{1}$$

where

$$|\mathcal{S};\rho_{[r]},\alpha_i\rangle \equiv |\mathcal{S};\rho_{[r]}\rangle|\mathcal{S};\alpha_i\rangle, \tag{2}$$

$$\widehat{\rho}|\mathcal{S};\rho_{[r]}\rangle = \rho_{[r]}|\mathcal{S};\rho_{[r]}\rangle, \tag{3}$$

$$\widehat{\sigma}_z|\mathcal{S};\alpha_i\rangle = \alpha_i|\mathcal{S};\alpha_i\rangle, \tag{4}$$

$$\alpha_1 = 1,\ \ \alpha_2 = -1. \tag{5}$$

Square brackets around subscripts denote indices of species eigenvalues. The species eigenvalues are nondegenerate but otherwise arbitrary:

$$\rho_{[1]} \neq \rho_{[2]}. \tag{6}$$

For later reference we note here the action of the spin operators in the x and y directions, $\widehat{\sigma}_x$ and $\widehat{\sigma}_y$, on $|\mathcal{S};\alpha_i\rangle$:

$$\begin{aligned}\widehat{\sigma}_x|\mathcal{S};\alpha_i\rangle &= |\mathcal{S};\alpha_{\bar{i}}\rangle, &(7)\\ \widehat{\sigma}_y|\mathcal{S};\alpha_i\rangle &= i\alpha_i|\mathcal{S};\alpha_{\bar{i}}\rangle, &(8)\end{aligned}$$

where $\bar{i}$ denotes the complement of i,

$$\bar{1} = 2,\ \ \bar{2} = 1. \tag{9}$$

2.1.2 Two-Particle Hilbert Space

The Hilbert space for two distinguishable particles is the tensor product of two single-particle Hilbert spaces, spanned by the eigenvectors

$$|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle \equiv |\mathcal{S}^{(1)};\rho_{[r]},\alpha_i\rangle|\mathcal{S}^{(2)};\rho_{[s]},\alpha_j\rangle \quad r,i,s,j=1,2, \tag{10}$$

where

$$\widehat{\rho}^{(1)}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle = \rho_{[r]}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle, \tag{11}$$

$$\widehat{\rho}^{(2)}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle = \rho_{[s]}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle, \tag{12}$$

$$\widehat{\sigma}_z^{(1)}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle = \alpha_i|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle. \tag{13}$$

$$\widehat{\sigma}_z^{(2)}|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle = \alpha_j|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle. \tag{14}$$

Superscript indices specifying particles are in parentheses.

2.1.3 Physical States

The physical two-particle states are the states in the two-particle Hilbert space above which are antisymmetric under interchange of particle index. This subspace is spanned by the (overcomplete) set of states

$$|[r],i,[s],j\rangle \equiv \frac{1}{\sqrt{2}}\left(|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\rho_{[s]},\alpha_j\rangle - |\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[s]},\alpha_j,\rho_{[r]},\alpha_i\rangle\right), \quad r,i,s,j=1,2. \tag{15}$$

We will be particularly interested in the four-dimensional subspace of physical states with one particle of each species.[4] We will use for this subspace the basis

$$|[1],i,[2],j\rangle, \quad i,j=1,2. \tag{16}$$

[4]Had we restricted ourselves to a single species, we would have at our disposal at this point only a single physical state, $(1/\sqrt{2})\left(|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle - |\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_1\rangle\right)$.

The operator which corresponds to a measurement of the total spin of the particles of species r is

$$\widehat{\vec{\sigma}}_{[r]} = |\mathcal{S}^{(1)};\rho_{[r]}\rangle\langle\mathcal{S}^{(1)};\rho_{[r]}| \otimes \widehat{\vec{\sigma}}^{(1)} \otimes \widehat{I}^{(2)} + \widehat{I}^{(1)} \otimes |\mathcal{S}^{(2)};\rho_{[r]}\rangle\langle\mathcal{S}^{(2)};\rho_{[r]}| \otimes \widehat{\vec{\sigma}}^{(2)} \tag{17}$$

The action of $\widehat{\sigma}_{[r],z}$ on the states (16) is

$$\widehat{\sigma}_{[r],z}|[1],i,[2],j\rangle = (\delta_{r1}\alpha_i + \delta_{r2}\alpha_j)\,|[1],i,[2],j\rangle. \tag{18}$$

So, spin and species are correlated in these states. States $|J,J_z\rangle$ of definite (spin) angular momentum can be constructed out of them:

$$\begin{aligned}
|1,1\rangle &= |[1],1,[2],1\rangle, && (19)\\
|1,0\rangle &= \frac{1}{\sqrt{2}}\Big(|[1],1,[2],2\rangle + |[1],2,[2],1\rangle\Big), && (20)\\
|0,0\rangle &= \frac{1}{\sqrt{2}}\Big(|[1],1,[2],2\rangle - |[1],2,[2],1\rangle\Big), && (21)\\
|1,-1\rangle &= |[1],2,[2],2\rangle, && (22)
\end{aligned}$$

where

$$\begin{aligned}
\widehat{\vec{j}}\cdot\widehat{\vec{j}}|J,J_z\rangle &= J(J+1)|J,J_z\rangle, && (23)\\
\widehat{j}_z|J,J_z\rangle &= J_z|J,J_z\rangle, && (24)
\end{aligned}$$

$\widehat{\vec{j}}$ being the total angular momentum operator,

$$\widehat{\vec{j}} = \left(\widehat{\vec{\sigma}}^{(1)} + \widehat{\vec{\sigma}}^{(2)}\right)/2. \tag{25}$$

2.1.4 Time Evolution and Entanglement

In [22] a system of two distinguishable particles, each with spin 1/2 and with no other dynamical properties, is considered. That is, the Hilbert space of the p^{th} particle is spanned by the two eigenvectors of the z component of its spin, $|\mathcal{S}^{(p)};\alpha_1\rangle$ and $|\mathcal{S}^{(p)};\alpha_2\rangle$.

These particles are prepared in the initial unentangled state

$$|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle, \tag{26}$$

where particle 1 has spin up and particle 2 has spin down. The particles are subsequently subjected to the action of the unitary time evolution operator $\widehat{u}_{E-dist}$ which, in the Schrödinger picture, has the effect of taking the state (26) to the entangled singlet state ($J = J_z = 0$):

$$\widehat{u}_{E-dist}|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle = \frac{1}{\sqrt{2}}\Big(|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle - |\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_1\rangle\Big) \tag{27}$$

The state (26) is considered unentangled because it is the product of a vector in the state space of particle 1 and a vector in the state space of particle 2. This notion of entanglement is inapplicable to the fermions with which we are concerned here, since any physical state of such particles must be antisymmetric under the interchange of the indices labeling the particles[25]. However, while it is not possible to distinguish particle 1 from particle 2, it it possbile to distinguish a particle of species 1 from a particle of species 2. We will therefore take the initial unentangled state of the two particles to be $|[1],1,[2],2\rangle$, containing a particle of species 1 with spin up and particle of species 2 with spin down. The time-evolution operator, $\widehat{u}_E$, will be such as to transform $|[1],1,[2],2\rangle$ to the singlet state (21); specifically,

$$\widehat{u}_E|[1],2,[2],1\rangle = |1,0\rangle \tag{28}$$

$$\widehat{u}_E|[1],1,[2],2\rangle = |0,0\rangle \tag{29}$$

$$\widehat{u}_E|[r],i,[s],j\rangle = |[r],i,[s],j\rangle, \quad \langle[1],2,[2],1|[r],i,[s],j\rangle = \langle[1],1,[2],2|[r],i,[s],j\rangle = 0. \tag{30}$$

This operator can be written as

$$\widehat{u}_E = \widehat{u}_E(\pi/4), \tag{31}$$

where

$$\begin{aligned}\widehat{u}_E(\gamma) &= \exp(-i\gamma\widehat{g}) \\ &= \widehat{I}' + \cos(\gamma)\widehat{I}_2 - i\sin(\gamma)\widehat{g}.\end{aligned} \tag{32}$$

Here $\widehat{I}_2$ is the projection operator into the two-dimensional subspace upon which $\widehat{u}_E$ acts nontrivially,

$$\widehat{I}_2 = |[1],2,[2],1\rangle\langle[1],2,[2],1| + |[1],1,[2],2\rangle\langle[1],1,[2],2|, \tag{33}$$

$\widehat{I}'$ is the projection operator into the rest of the state space,

$$\widehat{I}' = \widehat{I} - \widehat{I}_2, \tag{34}$$

($\widehat{I}$ = identity operator), and the generator of entanglement $\widehat{g}$ is given by

$$\widehat{g} = i\Big(|[1],1,[2],2\rangle\langle[1],2,[2],1| - |[1],2,[2],1\rangle\langle[1],1,[2],2|\Big). \tag{35}$$

2.1.5 Spin Correlation

The operator which corresponds to a measurement of the product of the projection of the spin of the species 1 particle along the unit vector $\vec{n}_1$ with the projection of the spin of the species 2 particle along the unit vector $\vec{n}_2$, is, using (17),

$$\begin{aligned}\widehat{\xi} &= (\vec{n}_1\cdot\widehat{\vec{\sigma}}_{[1]})(\vec{n}_2\cdot\widehat{\vec{\sigma}}_{[2]}) \\ &= |\mathcal{S}^{(1)};\rho_{[1]}\rangle\langle\mathcal{S}^{(1)};\rho_{[1]}|\otimes\vec{n}_1\cdot\widehat{\vec{\sigma}}^{(1)}\otimes\ |\mathcal{S}^{(2)};\rho_{[2]}\rangle\langle\mathcal{S}^{(2)};\rho_{[2]}|\otimes\vec{n}_2\cdot\widehat{\vec{\sigma}}^{(2)} \\ &\quad +|\mathcal{S}^{(1)};\rho_{[2]}\rangle\langle\mathcal{S}^{(1)};\rho_{[2]}|\otimes\vec{n}_2\cdot\widehat{\vec{\sigma}}^{(1)}\otimes\ |\mathcal{S}^{(2)};\rho_{[1]}\rangle\langle\mathcal{S}^{(2)};\rho_{[1]}|\otimes\vec{n}_1\cdot\widehat{\vec{\sigma}}^{(2)}.\end{aligned} \tag{36}$$

The spin correlation at the initial time t_0, when the particles are in the state $|[1],1,[2],2\rangle$, is

$$C_{1Q}(0) = \langle[1],1,[2],2|\,\widehat{\xi}\,|[1],1,[2],2\rangle. \tag{37}$$

Using (2)-(16) and (36),

$$C_{1Q}(0) = -n_{1,z}n_{2,z}. \tag{38}$$

In the state $\widehat{u}_E(\gamma)|[1], 1, [2], 2\rangle$,

$$C_{1Q}(\gamma) = \langle [1], 1, [2], 2|\, \widehat{u}_E^\dagger(\gamma)\, \widehat{\xi}\, \widehat{u}_E(\gamma)\, |[1], 1, [2], 2\rangle. \tag{39}$$

Using (2)-(16), (28)-(35) and (36),

$$C_{1Q}(\gamma) = -(1 - \sin(2\gamma))n_{1,z}n_{2,z} - \sin(2\gamma)\vec{n}_1 \cdot \vec{n}_2, \tag{40}$$

or, for small values of γ,

$$C_{1Q}(\gamma)_{|\gamma|\ll 1} = -(1 - 2\gamma)n_{1,z}n_{2,z} - 2\gamma\vec{n}_1 \cdot \vec{n}_2. \tag{41}$$

For $\gamma = \pi/4$,

$$C_{1Q}(\pi/4) = -\vec{n}_1 \cdot \vec{n}_2, \tag{42}$$

the correct value for distinguishable particles in the singlet state [26].

For present purposes we are taking 2γ, the degree to which the spin correlation is proportional to $-\vec{n}_1 \cdot \vec{n}_2$, as a heuristic measure of entanglement. This quantity is experimentally measurable, e.g., by performing many repetitions of the EPRB experiment with many different choices of $\vec{n}_1$ and $\vec{n}_2$ and determining the experimental value of 2γ by a least-squares fit to the data of the functional form (40) or (41). For a review of entanglement measures in general, see [27].

2.2 Fock Space Formalism

2.2.1 Correspondence with First-Quantized States and Operators

We consider a Fock space with four annihilation operators, corresponding to the four possible combinations of spin and species:

$$\widehat{\phi}_{[r]i}, \quad r, i = 1, 2, \tag{43}$$

where

$$\left\{\widehat{\phi}_{[r]i}, \widehat{\phi}_{[s]j}\right\} = \left\{\widehat{\phi}^\dagger_{[r]i}, \widehat{\phi}^\dagger_{[s]j}\right\} = 0, \tag{44}$$

$$\left\{\widehat{\phi}_{[r]i}, \widehat{\phi}^\dagger_{[s]j}\right\} = \delta_{rs}\delta_{ij}. \tag{45}$$

Curly brackets denote the anticommutator, $\left\{\widehat{A}, \widehat{B}\right\} = \widehat{A}\widehat{B} + \widehat{B}\widehat{A}$. The vacuum (no-particle) state $|0\rangle\rangle$ satisfies

$$\widehat{\phi}_{[r]i}\, |0\rangle\rangle = 0, \quad r, i = 1, 2, \tag{46}$$

The physical states of the two-particle Hilbert space correspond to states created from $|0\rangle\rangle$ by the creation-operator adjoints of (43):

$$|[r], i, [s], j\rangle \leftrightarrow |[r], i, [s], j\rangle\rangle, \tag{47}$$

where

$$|[r], i, [s], j\rangle\!\rangle = \widehat{\phi}^\dagger_{[r]i}\,\widehat{\phi}^\dagger_{[s]j}\,|0\rangle\!\rangle. \tag{48}$$

If $\widehat{\zeta}_1$ is an operator acting in the single-particle Hilbert space, the Fock space operator $\widehat{Z}_1$ corresponding to it,

$$\widehat{\zeta}_1 \leftrightarrow \widehat{Z}_1, \tag{49}$$

is

$$\widehat{Z}_1 = \sum_{r,i,s,j} \widehat{\phi}^\dagger_{[r]i}\,\zeta_{1,r,i,s,j}\,\widehat{\phi}_{[s]j}, \tag{50}$$

where

$$\zeta_{1,r,i,s,j} = \langle \mathcal{S}; [r], i|\,\widehat{\zeta}_1\,|\mathcal{S}; [s], j\rangle. \tag{51}$$

For example, the spin angular momentum is

$$\widehat{\vec{J}} = \frac{1}{2}\sum_{r,i,s,j} \widehat{\phi}^\dagger_{[r]i}\,\langle \mathcal{S}; [r], i|\,\widehat{\vec{\sigma}}\,|\mathcal{S}; [s], j\rangle\,\widehat{\phi}_{[s]j}. \tag{52}$$

Using (2)-(9), (44)-(48) and (52) we verify that

$$\begin{aligned}
|1,1\rangle\!\rangle &= |[1],1,[2],1\rangle\!\rangle, && (53)\\
|1,0\rangle\!\rangle &= \frac{1}{\sqrt{2}}\Big(|[1],1,[2],2\rangle\!\rangle + |[1],2,[2],1\rangle\!\rangle\Big), && (54)\\
|0,0\rangle\!\rangle &= \frac{1}{\sqrt{2}}\Big(|[1],1,[2],2\rangle\!\rangle - |[1],2,[2],1\rangle\!\rangle\Big), && (55)\\
|1,-1\rangle\!\rangle &= |[1],2,[2],2\rangle\!\rangle, && (56)
\end{aligned}$$

where

$$\begin{aligned}
\widehat{\vec{J}}\cdot\widehat{\vec{J}}|J, J_z\rangle\!\rangle &= J(J+1)|J, J_z\rangle\!\rangle, && (57)\\
\widehat{J}_z|J, J_z\rangle\!\rangle &= J_z|J, J_z\rangle\!\rangle. && (58)
\end{aligned}$$

If $\widehat{\zeta}_2$ is an operator in the two-particle Hilbert-space operator which mediates interactions between two particles, there is a corresponding Fock space operator $\widehat{Z}_2$:

$$\widehat{\zeta}_2 \leftrightarrow \widehat{Z}_2, \tag{59}$$

where

$$\widehat{Z}_2 = \frac{1}{2}\sum_{r',i',s',j'}\sum_{r,i,s,j} \widehat{\phi}^\dagger_{[s']j'}\,\widehat{\phi}^\dagger_{[r']i'}\,\zeta_{2,r',i',s',j';r,i,s,j}\,\widehat{\phi}_{[r]i}\,\widehat{\phi}_{[s]j}, \tag{60}$$

and

$$\zeta_{2,r',i',s',j';r,i,s,j} = \langle \mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r']}, \alpha_{i'}, \rho_{[s']}, \alpha_{j'}|\,\widehat{\zeta}_2\,|\mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r]}, \alpha_i, \rho_{[s]}, \alpha_j\rangle. \tag{61}$$

2.2.2 Time Evolution and Entanglement

The Fock space operator corresponding to the first quantized operator $\widehat{g}$ is, using (60) and (61),

$$\widehat{G} = \frac{1}{2} \sum_{r',i',s',j'} \sum_{r,i,s,j} \widehat{\phi}^\dagger_{[s']j'}\, \widehat{\phi}^\dagger_{[r']i'}\, g_{r',i',s',j';r,i,s,j}\, \widehat{\phi}_{[r]i}\, \widehat{\phi}_{[s]j}, \tag{62}$$

where

$$g_{r',i',s',j';r,i,s,j} \equiv \langle \mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r']}, \alpha_{i'}, \rho_{[s']}, \alpha_{j'} |\, \widehat{g}\, | \mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r]}, \alpha_i, \rho_{[s]}, \alpha_j \rangle \tag{63}$$

Using (2)-(16) and (35),

$$\begin{aligned} g_{r',i',s',j';r,i,s,j} &= (i/2) \Big[\left(\delta_{r'1}\delta_{i'1}\delta_{s'2}\delta_{j'2} - \delta_{r'2}\delta_{i'2}\delta_{s'1}\delta_{j'1}\right) \left(\delta_{r1}\delta_{i2}\delta_{s2}\delta_{j1} - \delta_{r2}\delta_{i1}\delta_{s1}\delta_{j2}\right) \\ &\quad - \left(\delta_{r',1}\delta_{i'2}\delta_{s'2}\delta_{j'1} - \delta_{r'2}\delta_{i'1}\delta_{s'1}\delta_{j'2}\right) \left(\delta_{r1}\delta_{i1}\delta_{s2}\delta_{j2} - \delta_{r2}\delta_{i2}\delta_{s1}\delta_{j1}\right) \Big]. \end{aligned} \tag{64}$$

The unitary operator

$$\widehat{U}_F(\gamma) = \exp(-i\gamma\widehat{G}) \tag{65}$$

acts on two-particle Fock space states in a manner corresponding to the action of $\widehat{u}_E$ on the two-particle physical Hilbert-space states:

$$\begin{aligned} &\widehat{U}_F(\gamma)\, \widehat{\phi}^\dagger_{[r]i}\, \widehat{\phi}^\dagger_{[s]j}\, |0\rangle\!\rangle = \\ &(1/2) \textstyle\sum_{r'i's'j'} \langle [r'], i', [s'], j' |\, \widehat{u}_E(\gamma)\, | [r], i, [s], j \rangle\, \widehat{\phi}^\dagger_{[r']i'}\, \widehat{\phi}^\dagger_{[s']j'}\, |0\rangle\!\rangle. \end{aligned} \tag{66}$$

In particular,

$$\widehat{U}_F | [1], 2, [2], 1 \rangle\!\rangle = | 1, 0 \rangle\!\rangle \tag{67}$$

$$\widehat{U}_F | [1], 1, [2], 2 \rangle\!\rangle = | 0, 0 \rangle\!\rangle \tag{68}$$

$$\begin{aligned} \widehat{U}_F | [r], i, [s], j \rangle\!\rangle &= | [r], i, [s], j \rangle\!\rangle, \\ &\langle\!\langle [1], 2, [2], 1 | [r], i, [s], j \rangle\!\rangle = \langle\!\langle [1], 1, [2], 2 | [r], i, [s], j \rangle\!\rangle = 0. \end{aligned} \tag{69}$$

where

$$\widehat{U}_F = \widehat{U}_F(\pi/4). \tag{70}$$

2.2.3 Spin Correlation

The Fock space equivalent of the first-quantized operator $\widehat{\xi}$ is

$$\widehat{\Xi}_F = \frac{1}{2} \sum_{r',i',s',j'} \sum_{r,i,s,j} \widehat{\phi}^\dagger_{[s']j'}\, \widehat{\phi}^\dagger_{[r']i'}\, \xi_{r',i',s',j';r,i,s,j}\, \widehat{\phi}_{[r]i}\, \widehat{\phi}_{[s]j}, \tag{71}$$

where

$$\xi_{r',i',s',j';r,i,s,j} \equiv \langle \mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r']}, \alpha_{i'}, \rho_{[s']}, \alpha_{j'} |\, \widehat{\xi}\, | \mathcal{S}^{(1)}\mathcal{S}^{(2)}; \rho_{[r]}, \alpha_i, \rho_{[s]}, \alpha_j \rangle. \tag{72}$$

Using (2), (6) and (36),

$$\begin{aligned} \xi_{r',i',s',j';r,i,s,j} = \; & \delta_{r'1}\delta_{s'2}\delta_{r1}\delta_{s2} \langle \mathcal{S}^{(1)}; \alpha_{i'} | \langle \mathcal{S}^{(2)}; \alpha_{j'} | (\vec{n}_1 \cdot \widehat{\vec{\sigma}}^{(1)})(\vec{n}_2 \cdot \widehat{\vec{\sigma}}^{(2)}) | \mathcal{S}^{(1)}; \alpha_i \rangle | \mathcal{S}^{(2)}; \alpha_j \rangle \\ & + \delta_{r'2}\delta_{s'1}\delta_{r2}\delta_{s1} \langle \mathcal{S}^{(1)}; \alpha_{i'} | \langle \mathcal{S}^{(2)}; \alpha_{j'} | (\vec{n}_2 \cdot \widehat{\vec{\sigma}}^{(1)})(\vec{n}_1 \cdot \widehat{\vec{\sigma}}^{(2)}) | \mathcal{S}^{(1)}; \alpha_i \rangle | \mathcal{S}^{(2)}; \alpha_j \rangle. \end{aligned} \tag{73}$$

Using (4), (5), (7)-(9) and (73), (71) becomes

$$\widehat{\Xi}_F = \sum_{i',j',i,j} \widehat{\phi}^\dagger_{[2]j'}\, \widehat{\phi}^\dagger_{[1]i'}\, \tilde{\xi}_{i',j',i,j}\, \widehat{\phi}_{[1]i}\, \widehat{\phi}_{[2]j} \tag{74}$$

where

$$\begin{aligned} \tilde{\xi}_{i',j',i,j} &= \langle \mathcal{S}^{(1)}; \alpha_{i'}|\langle \mathcal{S}^{(2)}; \alpha_{j'}|(\vec{n}_1 \cdot \widehat{\vec{\sigma}}^{(1)})(\vec{n}_2 \cdot \widehat{\vec{\sigma}}^{(2)})|\mathcal{S}^{(1)}; \alpha_i\rangle|\mathcal{S}^{(2)}; \alpha_j\rangle \\ &= \left[(n_{1,x} + in_{1,y}\alpha_i)\,\delta_{i'\bar{i}} + n_{1,z}\alpha_i\delta_{i'i}\right]\left[(n_{2,x} + in_{2,y}\alpha_j)\,\delta_{j'\bar{j}} + n_{2,z}\alpha_j\delta_{j'j}\right]. \end{aligned} \tag{75}$$

The spin correlation in the state $\widehat{U}_F(\gamma)\,|[1],1,[2],2\rangle\!\rangle$ is

$$C_F(\gamma) = \langle\!\langle [1],1,[2],2|\, \widehat{U}^\dagger_F(\gamma)\, \widehat{\Xi}_F\, \widehat{U}_F(\gamma)\,|[1],1,[2],2\rangle\!\rangle \tag{76}$$

Using (15), (32)-(35), (48), (64), (66), and (74)-(76), we find that, in agreement with the first-quantized results (40), (41),

$$C_F(\gamma) = -(1 - \sin(2\gamma))n_{1,z}n_{2,z} - \sin(2\gamma)\vec{n}_1 \cdot \vec{n}_2 \tag{77}$$

or, for small γ,

$$C_F(\gamma)_{|\gamma|\ll 1} = -(1 - 2\gamma)n_{1,z}n_{2,z} - 2\gamma\vec{n}_1 \cdot \vec{n}_2. \tag{78}$$

3 EPRB Entanglement in Quantum Field Theory

3.1 Operators and States

We now consider a quantum field theory, in three space dimensions, of a nonrelativistic spin 1/2 particle which comes in two distinct species. We work from the outset in the Heisenberg picture. The dynamical variables of the theory are time-dependent field operators defined at each point in space, $\widehat{\phi}_{[r]i}(\vec{x}, t)$, $r, i = 1, 2$. At the initial time t_0 these operators satisfy

$$\left\{\widehat{\phi}_{[r]i}(\vec{x}), \widehat{\phi}_{[s]j}(\vec{y})\right\} = \left\{\widehat{\phi}^\dagger_{[r]i}(\vec{x}), \widehat{\phi}^\dagger_{[s]j}(\vec{y})\right\} = 0, \tag{79}$$

$$\left\{\widehat{\phi}_{[r]i}(\vec{x}), \widehat{\phi}^\dagger_{[s]j}(\vec{y})\right\} = \delta_{rs}\delta_{ij}\delta^3(\vec{x} - \vec{y}), \tag{80}$$

where

$$\widehat{\phi}_{[r]i}(\vec{x}) \equiv \widehat{\phi}_{[r]i}(\vec{x}, t_0), \tag{81}$$

$$\widehat{\phi}^\dagger_{[r]i}(\vec{x}) \equiv \widehat{\phi}^\dagger_{[r]i}(\vec{x}, t_0). \tag{82}$$

By virtue of unitary time evolution, the equal-time anticommutation relations (79) and (80) are also satisfied at all later times t:

$$\left\{\widehat{\phi}_{[r]i}(\vec{x}, t), \widehat{\phi}_{[s]j}(\vec{y}, t)\right\} = \left\{\widehat{\phi}^\dagger_{[r]i}(\vec{x}, t), \widehat{\phi}^\dagger_{[s]j}(\vec{y}, t)\right\} = 0, \tag{83}$$

$$\left\{\widehat{\phi}_{[r]i}(\vec{x}, t), \widehat{\phi}^\dagger_{[s]j}(\vec{y}, t)\right\} = \delta_{rs}\delta_{ij}\delta^3(\vec{x} - \vec{y}), \tag{84}$$

The time-dependent Heisenberg-picture field operators act on time-independent states defined at $t = t_0$. For the vacuum state at t_0 we will use the same symbol as for the vacuum state of the Fock space of Sec. 2.2, $|0\rangle\rangle$:

$$\widehat{\phi}_{[r]i}(\vec{x})\,|0\rangle\rangle = 0, \qquad r, i = 1, 2, \tag{85}$$

An arbitrary normalized two-particle Heisenberg-picture state can be written as

$$|\psi\rangle\rangle = \sum_{r,i,s,j} \int d^3\vec{x}\, d^3\vec{y}\, \psi_{[r]i[s]j}(\vec{x}, \vec{y})\, \widehat{\phi}^\dagger_{[r]i}(\vec{x})\, \widehat{\phi}^\dagger_{[s]j}(\vec{x})\, |0\rangle\rangle \tag{86}$$

where $\psi_{[r]i[s]j}(\vec{x}, \vec{y})$ is a complex c-number function satisfying the normalization condition

$$\sum_{r,i,s,j} \int d^3\vec{x}\, d^3\vec{y}\, \psi^*_{[r]i[s]j}(\vec{x}, \vec{y})\, \psi_{[r]i[s]j}(\vec{x}, \vec{y}) = 1/2 \tag{87}$$

and, without loss of generality, the antisymmetry condition

$$\psi_{[r]i[s]j}(\vec{x}, \vec{y}) = -\psi_{[s]j[r]i}(\vec{y}, \vec{x}). \tag{88}$$

3.2 Hamiltonian and Exact Equation of Motion

The Hamiltonian is the sum of two parts,

$$\widehat{H} = \widehat{H}_0 + \epsilon \widehat{H}_1 \tag{89}$$

where ϵ is a small parameter. The free Hamiltonian is

$$\widehat{H}_0 = \sum_{r,i} \int d^3\vec{x}\, \widehat{\phi}^\dagger_{[r]i}(\vec{x}, t) \left(\frac{-\vec{\nabla}^2}{2m} \right) \widehat{\phi}_{[r]i}(\vec{x}, t). \tag{90}$$

For the interaction Hamiltonian we choose an interaction which, at each point $\vec{x}$, has the same form as the entanglement generator (62) of the Fock space of Sec. 2.2:

$$\widehat{H}_1 = \frac{1}{2} \sum_{r',i',s',j'} \sum_{r,i,s,j} \int d^3\vec{x}\, \kappa(\vec{x}, t)\, \widehat{\phi}^\dagger_{[s']j'}(\vec{x}, t)\, \widehat{\phi}^\dagger_{[r']i'}(\vec{x}, t)\, g_{r',i',s',j';r,i,s,j}\, \widehat{\phi}_{[r]i}(\vec{x}, t)\, \widehat{\phi}_{[s]j}(\vec{x}, t). \tag{91}$$

The possibility of spacetime dependence in the coupling, $\kappa(\vec{x}, t)$, has been allowed for.

Field operators evolve in time according to the Heisenberg equation of motion

$$\frac{\partial}{\partial t} \widehat{\phi}_{[r]i}(\vec{x}, t) = i[\widehat{H}, \widehat{\phi}_{[r]i}(\vec{x}, t)], \tag{92}$$

where we take $\hbar = 1$. Using the (83), (84), and (89)-(91) in (92), we obtain the exact equation of motion for $\widehat{\phi}_{[r]i}(\vec{x}, t)$,

$$\frac{\partial}{\partial t} \widehat{\phi}_{[r]i}(\vec{x}, t) = i \frac{\vec{\nabla}^2}{2m} \widehat{\phi}_{[r]i}(\vec{x}, t) + i\epsilon\kappa(\vec{x}, t) \sum_{s',j'} \sum_{q,k,s,j} g_{r,i,s',j';q,k,s,j}\, \widehat{\phi}^\dagger_{[s']j'}(\vec{x}, t)\, \widehat{\phi}_{[q]k}(\vec{x}, t)\, \widehat{\phi}_{[s]j}(\vec{x}, t). \tag{93}$$

3.3 First-order Perturbation Theory

We obtain a solution to $\widehat{\phi}_{[r]i}(\vec{x},t)$ using a straightforward perturbative approach (see, e.g., [28])in terms of the small parameter ϵ. We look for a solution of the form

$$\widehat{\phi}_{[r]i}(\vec{x},t)=\widehat{\phi}_{0,[r]i}(\vec{x},t)+\epsilon\widehat{\phi}_{1,[r]i}(\vec{x},t). \tag{94}$$

Using (94) in (93) we obtain the zeroth-order equation of motion

$$i\frac{\partial}{\partial t}\widehat{\phi}_{0,[r]i}(\vec{x},t)=-\frac{\vec{\nabla}^2}{2m}\widehat{\phi}_{0,[r]i}(\vec{x},t) \tag{95}$$

and the first-order equation of motion

$$i\frac{\partial}{\partial t}\widehat{\phi}_{1,[r]i}(\vec{x},t)=-\frac{\vec{\nabla}^2}{2m}\widehat{\phi}_{1,[r]i}(\vec{x},t)+\widehat{J}_{[r]i}(\vec{x},t), \tag{96}$$

where

$$\widehat{J}_{[r]i}(\vec{x},t)=-\kappa(\vec{x},t)\sum_{s',j'}\sum_{q,k,s,j}g_{r,i,s',j';q,k,s,j}\,\widehat{\phi}^{\dagger}_{0,[s']j'}(\vec{x},t)\,\widehat{\phi}_{0,[q]k}(\vec{x},t)\,\widehat{\phi}_{0,[s]j}(\vec{x},t). \tag{97}$$

We impose the initial condition

$$\widehat{\phi}_{1,[r]i}(\vec{x},t_0)=0, \tag{98}$$

so

$$\widehat{\phi}_{0,[r]i}(\vec{x},t_0)=\widehat{\phi}_{[r]i}(\vec{x},t_0)=\widehat{\phi}_{[r]i}(\vec{x}). \tag{99}$$

The solution for $\widehat{\phi}_{[r]i}(\vec{x},t)$ in terms of field operators at the initial time t_0 is

$$\widehat{\phi}_{[r]i}(\vec{x},t)=\int d^3\vec{x}' G(\vec{x}-\vec{x}',t-t_0)\widehat{\phi}_{[r]i}(\vec{x})-i\epsilon\int_{t_0}^{t}dt'\int d^3\vec{x}' G(\vec{x}-\vec{x}',t-t')\widehat{J}_{[r]i}(\vec{x}',t'), \tag{100}$$

where

$$\begin{aligned}\widehat{J}_{[r]i}(\vec{x},t) &= -\kappa(\vec{x},t)\textstyle\sum_{s',j'}\sum_{q,k,s,j}g_{r,i,s',j';q,k,s,j}\int d^3\vec{z}\,d^3\vec{z}'\,d^3\vec{z}''\\ & G^*(\vec{x}-\vec{z},t-t_0)G(\vec{x}-\vec{z}',t-t_0)G(\vec{x}-\vec{z}'',t-t_0)\,\widehat{\phi}^{\dagger}_{[s']j'}(\vec{z})\,\widehat{\phi}_{[q]k}(\vec{z}')\,\widehat{\phi}_{[s]j}(\vec{z}'').\end{aligned} \tag{101}$$

and $G(\vec{x}-\vec{x}',t-t')$ is the Schrödinger Green's function

$$G(\vec{x}-\vec{x}',t-t')=\left(\frac{-2mi}{4\pi(t-t')}\right)^{3/2}\exp\left(\frac{im|\vec{x}-\vec{x}'|^2}{2(t-t')}\right), \tag{102}$$

satisfying

$$\left(i\frac{\partial}{\partial t}+\frac{\vec{\nabla}^2}{2m}\right)G(\vec{x}-\vec{x}',t-t')=0 \tag{103}$$

and

$$G(\vec{x}-\vec{x}',0)=\delta^3(\vec{x}-\vec{x}'). \tag{104}$$

From (102) it follows that

$$G(\vec{x}'-\vec{x},t-t') = G(\vec{x}-\vec{x}',t-t'), \tag{105}$$

$$G(\vec{x}-\vec{x}',t'-t) = G^*(\vec{x}-\vec{x}',t-t'), \tag{106}$$

and

$$\int d^3\vec{x}''G(\vec{x}-\vec{x}'',t-t'')G^*(\vec{x}'-\vec{x}'',t'-t'')=G(\vec{x}-\vec{x}',t-t'). \tag{107}$$

3.4 Spin Correlation

For the initial-time Heisenberg-picture state we take a state, which we denote $|\psi_0\rangle\rangle$, containing one spin-up particle of species 1 and one spin-down particle of species 2:

$$\begin{aligned} \psi_{0,[1]1[2]2}(\vec{x},\vec{y}) &= -\psi_{0,[2]2[1]1}(\vec{y},\vec{x}) = \frac{1}{2}\,\psi_{[1]1}(\vec{x})\,\psi_{[2]2}(\vec{y}), \\ \psi_{0,[r]i[s]j}(\vec{x},\vec{y}) &= 0, \quad r,i,s,j \neq 1,1,2,2 \text{ or } 2,2,1,1, \end{aligned} \tag{108}$$

where

$$\int d^3\vec{x}\,\psi^*_{[1]1}(\vec{x})\,\psi_{[1]1}(\vec{x}) = \int d^3\vec{x}\,\psi^*_{[2]2}(\vec{x})\,\psi_{[2]2}(\vec{x}) = 1. \tag{109}$$

That is,

$$|\psi_0\rangle\rangle = \int d^3\vec{x}\,d^3\vec{y}\,\psi_{[1]1}(\vec{x})\,\psi_{[2]2}(\vec{y})\,\widehat{\phi}^\dagger_{[1]1}(\vec{x})\,\widehat{\phi}^\dagger_{[2]2}(\vec{y})\,|0\rangle\rangle. \tag{110}$$

The field-theoretic operator corresponding to the first-quantized spin-correlation operator (36) is

$$\begin{aligned} \widehat{\Xi}(t) &= \frac{1}{2}\int d^3\vec{x}'\,d^3\vec{y}'\,d^3\vec{x}\,d^3\vec{y}\sum_{r',i',s',j'}\sum_{r,i,s,j} \\ &\quad \widehat{\phi}^\dagger_{[s']j'}(\vec{y}',t)\,\widehat{\phi}^\dagger_{[r']i'}(\vec{x}',t)\,\xi_{r',i',s',j';r,i,s,j}(\vec{x}',\vec{y}',\vec{x},\vec{y})\,\widehat{\phi}_{[r]i}(\vec{x},t)\,\widehat{\phi}_{[s]j}(\vec{y},t), \end{aligned} \tag{111}$$

where

$$\xi_{r',i',s',j';r,i,s,j}(\vec{x}',\vec{y}',\vec{x},\vec{y}) = \langle \mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r']},\alpha_{i'},\vec{x}',\rho_{[s']},\alpha_{j'},\vec{y}'|\,\widehat{\xi}\,|\mathcal{S}^{(1)}\mathcal{S}^{(2)};\rho_{[r]},\alpha_i,\vec{x},\rho_{[s]},\alpha_j,\vec{y}\rangle. \tag{112}$$

Since $\widehat{\xi}$ acts trivially on spatial coordinates,

$$\xi_{r',i',s',j';r,i,s,j}(\vec{x}',\vec{y}',\vec{x},\vec{y}) = \xi_{r',i',s',j';r,i,s,j}\delta^3(\vec{x}'-\vec{x})\delta^3(\vec{y}'-\vec{y}), \tag{113}$$

where $\xi_{r',i',s',j';r,i,s,j}$ is as given in (73). Using (73) and (75), (111) becomes

$$\widehat{\Xi}(t) = \int d^3\vec{x}\,d^3\vec{y}\sum_{i',j',i,j}\widehat{\phi}^\dagger_{[2]j'}(\vec{y},t)\,\widehat{\phi}^\dagger_{[1]i'}(\vec{x},t)\,\tilde{\xi}_{i',j',i,j}\,\widehat{\phi}_{[1]i}(\vec{x},t)\,\widehat{\phi}_{[2]j}(\vec{y},t). \tag{114}$$

The spin correlation at time t given the initial state $|\psi_0\rangle\rangle$ is

$$C(t) = \langle\langle\psi_0|\,\widehat{\Xi}(t)\,|\psi_0\rangle\rangle. \tag{115}$$

Using (64), (75), (79), (80), (85), (100), (101), (104)-(107), (110) and (114) we find

$$C(t) = -(1-\epsilon L(t))n_{1,z}n_{1,z} - \epsilon L(t)\vec{n}_1\cdot\vec{n}_2 \tag{116}$$

where

$$\begin{aligned} L(t) = \;& 2\textstyle\int_{t_0}^t dt''\int d^3\vec{x}''\kappa(\vec{x}'',t'') \\ & \left[\textstyle\int d^3\vec{y}'\,\psi^*_{[2]2}(\vec{y}')\,G^*(\vec{y}'-\vec{x}'',t''-t_0)\right]\left[\textstyle\int d^3\vec{x}'\,\psi^*_{[1]1}(\vec{x}')\,G^*(\vec{x}'-\vec{x}'',t''-t_0)\right] \\ & \left[\textstyle\int d^3\vec{x}\,\psi_{[1]1}(\vec{x})\,G(\vec{x}-\vec{x}'',t''-t_0)\right]\left[\textstyle\int d^3\vec{y}\,\psi_{[2]2}(\vec{y})\,G(\vec{y}-\vec{x}'',t''-t_0)\right]. \end{aligned} \tag{117}$$

3.5 Gaussian Wavepackets

Now suppose that the functions $\psi_{[1]1}(\vec{x})$, $\psi_{[2]2}(\vec{x})$, are of the form

$$\psi_{[1]1}(\vec{x}) = \psi_g(\vec{x}_1, \vec{v}_1; \vec{x}) \tag{118}$$

$$\psi_{[2]2}(\vec{x}) = \psi_g(\vec{x}_2, \vec{v}_2; \vec{x}) \tag{119}$$

where

$$\psi_g(\vec{x}_i, \vec{v}_i; \vec{x}) = \left(\frac{\alpha}{\pi}\right)^{3/4} \exp\left(-\frac{\alpha|\vec{x}-\vec{x}_i|^2}{2} + im\vec{v}_i \cdot (\vec{x}-\vec{x}_i)\right). \tag{120}$$

That is, the initial state consists of two Gaussian wavepackets each of which, at time t_0, has width $1/\sqrt{\alpha}$, the wavepacket for the particle of species i being centered at position $\vec{x}_i$ and moving with velocity $\vec{v}_i$. Using (102) and (118)-(120), $L(t)$ in eq. (117) in this case has the value

$$L_g(t) = 2\int_{t_0}^{t} dt'' \left(\frac{A(t'')}{\pi}\right)^3 \int d^3\vec{x}''\kappa(\vec{x}'', t'') \exp\left(-A(t'')\left[|\vec{x}''-\vec{c}_1(t'')|^2 + |\vec{x}''-\vec{c}_2(t'')|^2\right]\right), \tag{121}$$

where

$$A(t'') = \frac{\alpha}{1+\alpha^2\left(t''-t_0\right)^2/m^2} \tag{122}$$

and

$$\vec{c}_i(t'') = \vec{x}_i + \vec{v}_i(t''-t_0), \quad i = 1, 2. \tag{123}$$

$\vec{c}_i(t'')$ is the trajectory of the center of the wavepacket of the particle of species i.

3.5.1 Point Interaction

Suppose that the entangling interaction between particles of different species is only "turned on" in a very limited region of space, idealized as a point $\vec{x}_I$, and only for a brief extent of time idealized as a single moment t_I:

$$\kappa(\vec{x}, t'') = \kappa\delta^3(\vec{x}-\vec{x}_I)\delta(t''-t_I). \tag{124}$$

$L_g(t)$ in (121) then has the value

$$L_{g-PI}(t) = 2\kappa\left(\frac{A(t_I)}{\pi}\right)^3 \exp\left(-A(t_I)\left[|\vec{x}_I-\vec{c}_1(t_I)|^2 + |\vec{x}_I-\vec{c}_2(t_I)|^2\right]\right). \tag{125}$$

This quantity will be exponentially small unless

$$\vec{c}_1(t_I) \approx \vec{c}_2(t_I) \approx \vec{x}_I. \tag{126}$$

That is, there will be little entanglement generated unless the initial locations and velocities of the centers of the wavepackets are such as to arrive near the point $\vec{x}_I$ very close to the time t_I.

3.5.2 Position-Independent Interaction

Consider now that the coupling $\kappa(\vec{x}, t'')$ is independent of position but still possibly time-dependent:

$$\kappa(\vec{x}, t'') = \kappa(t''). \tag{127}$$

Using this in (121), $L_g(t)$ becomes

$$L_{g-PII} = \frac{1}{\sqrt{2}} \int_{t_0}^{t} dt'' \kappa(t'') \left(\frac{A(t'')}{\pi} \right)^{3/2} \exp\left(-\frac{1}{2} A(t'') |\vec{c}_1(t'') - \vec{c}_2(t'')|^2 \right) \tag{128}$$

We can approximate the integral in (128) by the method of steepest descent (see, e.g., [29]),

$$\int dt''\, f(t'') \approx \sqrt{\frac{2\pi f(t_c)^3}{-\ddot{f}(t_c)}}, \tag{129}$$

where t_c is the point at which the integrand $f(t'')$ attains its maximum ($\dot{f}(t_c) = 0$). The approximation is better the more the integrand peaks at one value of the integration variable. Therefore, taking into account (122) and (128), we consider the limit in which the width $1/\sqrt{\alpha}$ of each wavepacket becomes small and the mass m of each particle becomes large in such a way that

$$\frac{\alpha^2 (t - t_0)^2}{m^2} \to 0. \tag{130}$$

In this limit

$$A(t'') \approx \alpha \tag{131}$$

so

$$L(t)_{g-PII} \approx \frac{1}{\sqrt{2}} \left(\frac{\alpha}{\pi} \right)^{3/2} \int_{t_0}^{t} dt'' \mathcal{L}(t'') \tag{132}$$

where

$$\mathcal{L}(t'') = \kappa(t'') \exp\left(-\frac{\alpha}{2} |\vec{c}_1(t'') - \vec{c}_2(t'')|^2 \right). \tag{133}$$

Setting $\dot{\mathcal{L}}(t_c) = 0$ we find, using (123) and (133),

$$t_c = t_0 + \frac{1}{\alpha |\vec{v}_1 - \vec{v}_2|^2} \left[\frac{\dot{\kappa}(t_c)}{\kappa(t_c)} - \alpha (\vec{v}_1 - \vec{v}_2) \cdot (\vec{x}_1 - \vec{x}_2) \right] \tag{134}$$

If the coupling $\kappa(t'')$ changes sufficiently smoothly with time in the vicinity of t_c—specifically,

$$|\dot{\kappa}(t_c)/\kappa(t_c)| \ll \alpha (\vec{v}_1 - \vec{v}_2) \cdot (\vec{x}_1 - \vec{x}_2) \tag{135}$$

and

$$|\ddot{\kappa}(t_c)/\kappa(t_c)| \ll \alpha |\vec{v}_1 - \vec{v}_2|^2 \tag{136}$$

—then

$$t_c \approx t_{min} = t_0 - \frac{(\vec{v}_1 - \vec{v}_2) \cdot (\vec{x}_1 - \vec{x}_2)}{|\vec{v}_1 - \vec{v}_2|^2} \tag{137}$$

where t_{min} is the time at which the centers of the two wavepackets attain their minimum separation d_{min},

$$d_{min} = \min_{t''} |\vec{c}_1(t'') - \vec{c}_2(t'')|, \tag{138}$$

$$t_{min} = \arg\min |\vec{c}_1(t'') - \vec{c}_2(t'')|, \tag{139}$$

and the degree of entanglement at time t is

$$L(t)_{g-PII} \approx \frac{\alpha\kappa(t_{min})}{\pi}\sqrt{\frac{1}{|\vec{v}_1 - \vec{v}_2|^2}}\exp\left(-\frac{\alpha}{2}d_{min}^2\right). \tag{140}$$

This result is again intuitively reasonable. The degree of entanglement at the final time t is an exponentially-decreasing function of the "miss distance" of the centers of the two Gaussian wavepackets, and is inversely proportional to the speed $|\vec{v}_1 - \vec{v}_2|$ with which they pass each other (lower speed providing more time to interact while the wavepackets most nearly overlap.)

4 The Vacuum Representation

At first glance, the results of the previous section for the expected values of the spin correlation $C(t)$ may well seem to fit with a conceptual picture of entanglement being generated and then carried along to measuring devices in a local manner, as operator-valued wavepackets encounter each other and then proceed on to the experimenters' apparatus. But, in fact, the formalism presented above nowhere displays a local mechanism for transporting the information $\vec{x}_i$, $\vec{v}_i$, about the initial conditions at time t_0 to later times. Whether we examine the exact equation of motion (93) for $\widehat{\phi}(\vec{x},t)$ or the perturbative solution (100), (101), we see that—despite the fact that $\widehat{\phi}(\vec{x},t)$ is expressed in the latter in terms of the time-t_0 operators $\widehat{\phi}(\vec{x})$, $\widehat{\phi}^\dagger(\vec{x})$—$\widehat{\phi}(\vec{x},t)$ is completely independent, at any time t, of the information contained in the initial state $|\psi_0\rangle\rangle$.

In the Schrödinger picture, information on initial conditions and time evolution is carried by the state vector. In the Heisenberg picture, information on time evolution is transfered to the operators, but information on initial conditions continues to reside in the state vector. In the usual Heisenberg-picture quantum field theory, this results in the situation that "the quantized solution to a field equation is not *a* solution, it is in some sense *the* solution . . . A 'quantum field' corresponds more closely to a 'general solution' to a field equation than to a specific solution reflecting complete initial and boundary values. As the analogue of a classical 'general solution,' we again describe the 'operator-valued quantum field' most accurately as a field determinable, something that charts the spatiotemporal relations of any of a large set of possible field configurations [30]."(See also [31], p. 1897.)

What is necessary to see explicitly the locality of Heisenberg-picture quantum field theory is to complete the transfer of information from states to operators. As pointed out by Deutsch and Hayden [21] in the context of Heisenberg-picture quantum mechanics, this can be accomplished by performing a unitary transformation which takes the Heisenberg-picture state vector, containing the initial-condition information, to some standard state vector which is the same regardless of initial conditions. To keep the physics unchanged, the same unitary transformation must of course be applied to the operators. In such a representation, "the term 'state vector' becomes a misnomer, for [it] contains no information ... All the information is contained in the observables [21]."

For field theory a natural choice for the standard state is the vacuum state $|0\rangle\!\rangle$. Consider the operator

$$\widehat{V}[\psi_{[1]1},\psi_{[2]2}](\theta) \equiv \exp(\theta \widehat{W}[\psi_{[1]1},\psi_{[2]2}]), \tag{141}$$

θ real, where

$$\begin{aligned}&\widehat{W}[\psi_{[1]1},\psi_{[2]2}] \equiv \\ &\quad \int d^3\vec{x}\, d^3\vec{y}\, \left(\psi_{[1]1}(\vec{x})\, \psi_{[2]2}(\vec{y})\, \widehat{\phi}^{\dagger}_{[1]1}(\vec{x})\, \widehat{\phi}^{\dagger}_{[2]2}(\vec{y}) - \psi^*_{[1]1}(\vec{x})\, \psi^*_{[2]2}(\vec{y})\, \widehat{\phi}_{[2]2}(\vec{y})\, \widehat{\phi}_{[1]1}(\vec{x})\right).\end{aligned} \tag{142}$$

The square brackets here denote functional dependence; for notational convenience this will not be explicitly indicated below. $\widehat{W}$ is anti-Hermitian, so $\widehat{V}(\theta)$ is unitary. Using (79), (80), (85), (142) and mathematical induction, we find that

$$\widehat{W}^{2n}|0\rangle\!\rangle = (-1)^n|0\rangle\!\rangle, \qquad n = 0, 1, 2, \ldots, \tag{143}$$

$$\widehat{W}^{2n+1}|0\rangle\!\rangle = (-1)^n|\psi_0\rangle\!\rangle \qquad n = 0, 1, 2, \ldots, \tag{144}$$

so

$$\widehat{V}(\theta)|0\rangle\!\rangle = \cos(\theta)|0\rangle\!\rangle + \sin(\theta)|\psi_0\rangle\!\rangle. \tag{145}$$

Defining

$$\widehat{V} = \widehat{V}(\pi/2), \tag{146}$$

we see that

$$\widehat{V}|0\rangle\!\rangle = |\psi_0\rangle\!\rangle, \tag{147}$$

$$\widehat{V}^{\dagger}|\psi_0\rangle\!\rangle = |0\rangle\!\rangle. \tag{148}$$

We can therefore transform to a "vacuum representation" in which the Heisenberg-picture state vector is simply $|0\rangle\!\rangle$ and the initial-time operators, $\widehat{\phi}_{V,[r]i}(\vec{x})$, depend, through $\widehat{V}$, on the information which, in the usual representation, is contained in $|\psi_0\rangle\!\rangle$. That is,

$$\widehat{\phi}_{V,[r]i}(\vec{x}) = \widehat{V}^{\dagger}\, \widehat{\phi}_{[r]i}(\vec{x})\, \widehat{V}. \tag{149}$$

Using (141), (146) and (149)

$$\begin{aligned}\widehat{\phi}_{V,[r]i}(\vec{x}) = \widehat{\phi}_{[r]i}(\vec{x}) + \frac{\pi}{2}[\widehat{\phi}_{[r]i}(\vec{x}),\widehat{W}] + \frac{1}{2!}\left(\frac{\pi}{2}\right)^2[[\widehat{\phi}_{[r]i}(\vec{x}),\widehat{W}],\widehat{W}] \\ + \frac{1}{3!}\left(\frac{\pi}{2}\right)^3[[[\widehat{\phi}_{[r]i}(\vec{x}),\widehat{W}],\widehat{W}],\widehat{W}] + \ldots\ .\end{aligned} \tag{150}$$

(See, e.g., [32].) Brackets denote the commutator, $[\widehat{A},\widehat{B}] = \widehat{A}\widehat{B} - \widehat{B}\widehat{A}$. Using (79), (80) and (142),

$$[\widehat{\phi}_{[r]i}(\vec{x}),\widehat{W}] = \delta_{r1}\delta_{i1}\psi_{[1]1}(\vec{x})\int d^3\vec{y}\psi_{[2]2}(\vec{y})\widehat{\phi}^{\dagger}_{[2]2}(\vec{y}) - \delta_{r2}\delta_{i2}\psi_{[2]2}(\vec{x})\int d^3\vec{y}\psi_{[1]1}(\vec{y})\widehat{\phi}^{\dagger}_{[1]1}(\vec{y}). \tag{151}$$

From (150) and (151), we see that $\widehat{\phi}_{V,[r]i}(\vec{x})$ is identical to $\widehat{\phi}_{[r]i}(\vec{x})$ at any location $\vec{x}$ where $\psi_{[r]i}(\vec{x})$ vanishes:

$$\widehat{\phi}_{V,[r]i}(\vec{x}) = \widehat{\phi}_{[r]i}(\vec{x}), \quad \vec{x} \notin \text{support of } \psi_{[r]i}. \tag{152}$$

Since $\widehat{V}$ is independent of time, the equation of motion (93) retains its form

$$\frac{\partial}{\partial t}\widehat{\phi}_{V,[r]i}(\vec{x},t) = i\frac{\vec{\nabla}^2}{2m}\widehat{\phi}_{V,[r]i}(\vec{x},t)$$
$$+i\epsilon\kappa(\vec{x},t)\sum_{s',j'}\sum_{q,k,s,j} g_{r,i,s',j';q,k,s,j}\,\widehat{\phi}^{\dagger}_{V,[s']j'}(\vec{x},t)\,\widehat{\phi}_{V,[q]k}(\vec{x},t)\,\widehat{\phi}_{V,[s]j}(\vec{x},t), \tag{153}$$

as does the first-order perturbative solution (100), (101):

$$\begin{aligned}\widehat{\phi}_{V,[r]i}(\vec{x},t) &= \int d^3\vec{x}' G(\vec{x}-\vec{x}', t-t_0)\widehat{\phi}_{V,[r]i}(\vec{x}) \\ &\quad -i\epsilon\int_{t_0}^{t} dt'\int d^3\vec{x}' G(\vec{x}-\vec{x}', t-t')\widehat{J}_{V,[r]i}(\vec{x}',t'),\end{aligned} \tag{154}$$

$$\widehat{J}_{V,[r]i}(\vec{x},t) = -\kappa(\vec{x},t)\textstyle\sum_{s',j'}\sum_{q,k,s,j} g_{r,i,s',j';q,k,s,j}\int d^3\vec{z}\, d^3\vec{z}'\, d^3\vec{z}''$$
$$G^*(\vec{x}-\vec{z}, t-t_0)G(\vec{x}-\vec{z}', t-t_0)G(\vec{x}-\vec{z}'', t-t_0)\,\widehat{\phi}^{\dagger}_{V,[s']j'}(\vec{z})\,\widehat{\phi}_{V,[q]k}(\vec{z}')\,\widehat{\phi}_{V,[s]j}(\vec{z}''), \tag{155}$$

where, of course,

$$\widehat{\phi}^{\dagger}_{V,[r]i}(\vec{x},t) = \widehat{V}^{\dagger}\,\widehat{\phi}^{\dagger}_{[r]i}(\vec{x},t)\,\widehat{V}, \tag{156}$$

$$\widehat{J}_{V,[r]i}(\vec{x},t) = \widehat{V}^{\dagger}\,\widehat{J}_{[r]i}(\vec{x},t)\,\widehat{V}, \tag{157}$$

etc.

In these expressions the manner in which information is encoded in the operators at t_0 and subsequently transferred, in accordance with the rules of a local differential equation, to operators at later times, is evident. So, at least from a conceptual point of view, one should regard this representation as the fundamental one. In principle, one could even work in this representation to compute quantities such as the spin correlation $C(t)$ in (115). However, since the values of matrix elements are not changed by unitary transformations—in particular,

$$\langle\langle 0|\,\widehat{\Xi}_V(t)\,|0\rangle\rangle = \langle\langle\psi_0|\,\widehat{\Xi}(t)\,|\psi_0\rangle\rangle \tag{158}$$

—it is not necessary to do so. The usual representation of Heisenberg-picture quantum field theory, with initial-condition information contained in the state vector, may thus be regarded as an auxiliary representation to which one transforms from the explicitly-local vacuum representation for purposes of computational convenience.

5 Discussion

We see that the same mechanism that brings about correlations at a distance in Heisenberg-picture quantum mechanics in a local manner also works in Heisenberg-picture quantum field theory, at least in the nonrelativistic fermionic case. In the vacuum representation of Sec. 4, or in any other representation in which the state vector is independent of initial conditions, all information regarding initial values of physical quantities is encoded in the initial-time field operators. Information encoded in an operator at one place at an earlier time is transfered to operators at other places at later times in accordance with a local differential equation. (In a relativistic theory information is only transferred to later-time operators within the future light cone of the earlier-time operator.) Operator-valued wave packets corresponding to initially-separated particles may come into contact and exchange information. At any time, at any location, the value of the field operator is a weighted sum of products of initial-time field operators, as in (154), (155) (of course higher-order terms will in general be present). As one wave packet passes by another it may acquire contributions to this weighted sum which were "carried" to the interaction region by another wave packet corresponding to another particle. At later times operators in this wave packet will retain these contributions, serving as labels indicating that the encounter with the other wave packet took place. Depending on the nature of the initial conditions and the interaction, distant field operators at times subsequent to the interaction may be entangled in such a way that the results of measurements made upon them (when compared at still later times by means of some other causal interaction) are correlated to a degree in excess of that allowed by Bell's theorem.[5] Since, in the Everett interpretation, correlations are correlations of information exchanged in a causal manner between copies of measuring instruments and/or the states of awareness of observers, these excess correlations in no way imply the presence of nonlocality.

Since the issue of locality or the lack thereof is generally framed in the EPRB experiment in terms of particles which are distinguishable, at least by virtue of their spatial locations, and since the goal of the present paper has been to understand how the mechanism which permits locality in the first-quantized theory operates in the field-theoretic formalism, the focus above has been on states consisting of particles of distinct species, which can always in principle be distinguished. This is not to say that the issue of indistinguishability, although distinct from that of locality, does not impact locality.[6] Indeed, it is difficult to see how the labeling mechanism would operate causally in the context of symmetrized or antisymmetrized first-quantized Hilbert space. In field theory, as seen above, it is the field operators which are labeled by transformations induced by interactions, so there is no problem in this regard.

As for the label-proliferation problem, quantum field theory provides no explanation of the manner in which this information is recorded. The representation of the label information does seems more natural in quantum field theory than in point-particle quantum mechanics. In quantum mechanics, the operators pertaining to each particle acquire tensor-product factors acting in the state spaces of other particles with which the particle in question interacts. In quantum field theory, each operator acts in an infinite-dimensional space (see, e.g., [49] for an explicit representation) and changes the nature of its action in this space based on the nature of nearby operators. But, be it quantum-mechanical or quantum-field-theoretic, a single quantum operator is capable of carrying an unlimited amount of information regarding past interactions.[7]

[5] Entanglement and violation of Bell's theorem are related but not equivalent phenomena [33]

[6] The connection between entanglement and particle indistinguishability is examined in [34]-[48]

[7] Kent [50] presents a hidden-variable model in which "every hidden particle trajectory carries a record of its past history—which particles it was originally entangled with, and which measurements were carried

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out on it—arbitrarily far into the future." This is precisely the information which, according to quantum theory *sans* hidden variables, particles do indeed carry with them. See also [51].

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